Hadamard Inverse Function Theorem Proved by Variational Analysis

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We present a proof of Hadamard Inverse Function Theorem by the methods of Variational Analysis, adapting an idea of I. Ekeland and E. Séré [1].

Recall that Hadamard Inverse Function Theorem states:

Let $f \in C^1$, $f'(x)$ be invertible for all x and satisfying

$$\|[f'(x)]^{-1}\| \leq M, \quad \forall x \in X, \tag{1}$$

for some $M > 0$.

Then f is C^1 invertible on X.

In other words, there is $g \in C^1$ such that

$$g(f(x)) = f(g(x)) = x, \quad \forall x \in X.$$

References

[1] I. Ekeland and E. Séré, A local surjection theorem, 2017, https://project.inria.fr/brenier60/files/2011/12/Brenier.pdf

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